5.16 Euler method for differential equations

Consider the differential equation

$$\frac{dy}{dx} = x - y^2,$$

with initial condition $y(1.3) = 2.35$. Suppose we want to approximate the value of $y(2.2)$ rounded up to four significant figures, and using steps of size $h = 0.1$.

5.16.1 Input the sequences

We want to iterate over the $x's$ and the $y's$. The Euler method tells us to choose the sequences defined by

$$x_{n+1} = x_n + 0.1 \tag{1}$$

$$y_{n+1} = y_n + 0.1 \cdot (x_n - y_n^2) \tag{2}$$

The initial condition tells us that we start with x_1 and y_1. Here, $x_1 = 1.3$ and $y_1 = 2.35$ (because $y(x_1) = 2.35$).

> In the TI-Nspire, sequences are written $u(n)$ and $v(n)$ and not x_n and y_n. Thus in the following lines "$u(n)$" will mean " x_n" (so for example $u(2) = x_2 = 1.4$), and "$v(n)$" will mean "y_n".

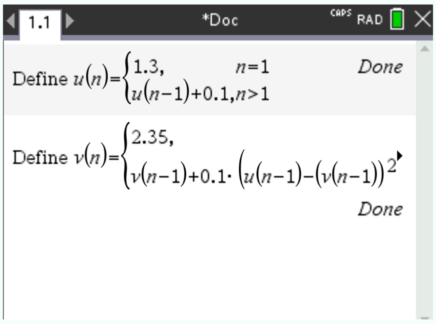
① Create a new document, press  and select Add Calculator.

② Press  , select Actions > Define.

③ Type $u(n) =$, press  and select 

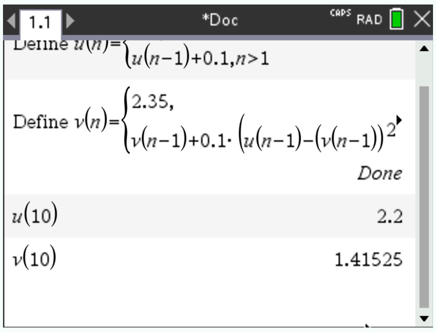
④ On the first line, write the initialization. On the second line, write the recursive expression.

⑤ Do the same for $v(n)$



5.16.2 Find the result asked

Since it is asked to compute $y(2.2)$, we read $u(10) = 2.2$ (recall that $u(10)$ is x_{10}), so our answer is $v(10) = 1.42$ (since $v(10)$ is y_{10} and Euler method tells us that $y_{10} \approx y(x_{10})$):



Thus, $y(2.2) \approx 1.42$.