2.4 Key features of graphs

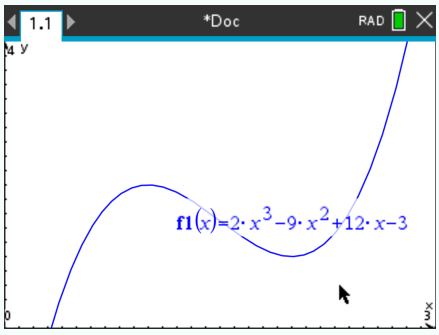
2.4.1 Find the maximum or minimum of a function, and its range

Find a local maximum or minimum

Suppose you want to know the local maximum(s) and minimum(s) of the following function:

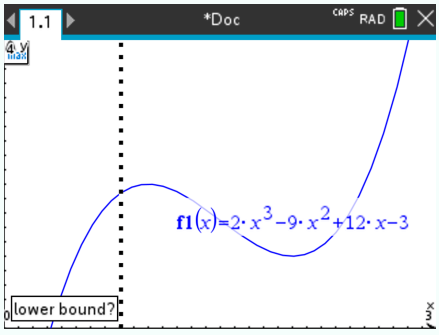
$$f(x) = 2x^3 - 9x^2 + 12x - 3$$

① Enter the function and graph it properly. Here we choose **Xmin**= 0, **Xmax**= 3, **Ymin**= 0, **Ymax**= 4:



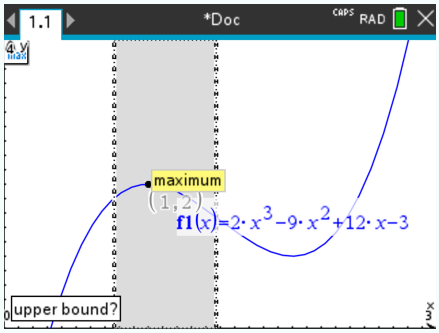
② Press menu and select Analyze Graph > Maximum.

③ Choose a left bound (as asked at the bottom of the screen) until you reach the left of the local maximum:



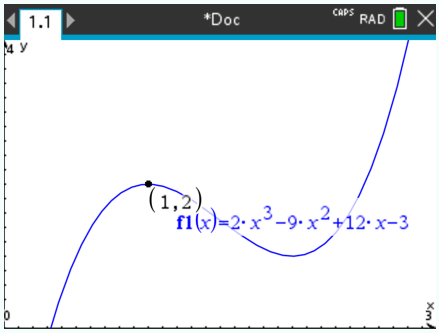
Press enter

④ Choose a right bound (as asked at the bottom of the screen):



Press enter

⑤ Read the maximum:



Press enter

The result should be $x = 1$ and $y = 2$ (it is displayed next to the point).

You can apply the same procedure to find a local minimum by pressing select Analyze Graph > Minimum instead of select Analyze Graph > Maximum.

Find together the range, the global maximum and the global minimum of a function within a domain

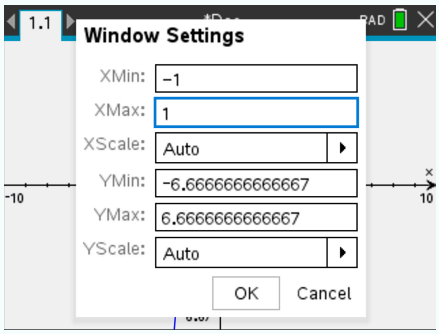
> This strategy is useful when one one only wants the y values of the global maximum or minimum. If you have to know the x values too, see 2.4.1

Suppose you want to find the maximum and minimum of the following function:

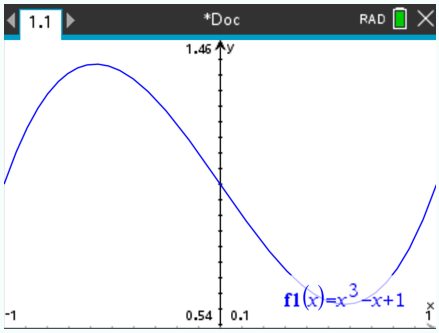
$$f(x) = x^3 - x + 1, \quad -1 \leq x \leq 1.$$

① Input the function

② Press menu and select Window / Zoom > Window Settings and fill **Xmin** and **Xmax** accordingly to the domain of the function:

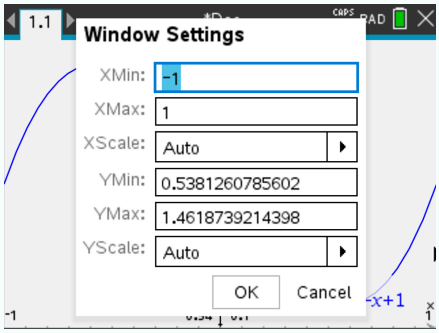


③ Press menu and select Window / Zoom > Zoom - Fit, and wait. The following should display:



Zoom-fit fits the y axis to match the range of f

④ Press menu and select Window / Zoom > Window Settings again, and read the values of **Ymin** and **Ymax**:



Thus, the minimum of f is 0.538, and the maximum is 1.462. Thus, the range of f is

$$0.538 \leq y \leq 1.462.$$

2.4.2 Find the $y-$intercept of a function

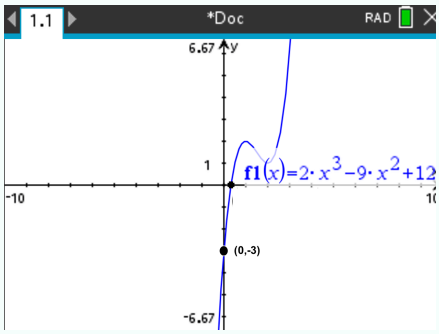
Suppose you want to know the $y-$intercept of the following function:

$$f(x) = 2x^3 - 9x^2 + 12x - 3$$

Recall that we want to compute $f(0)$.

① Enter the function, and display its graph.

② Press menu and select Trace > Graph Trace. Press enter and it automatically shows the $y-$intercept.



The result **Y=-3** should be displayed within the coordinate of the point (at the bottom right of the screen).

2.4.3 Find one of the $x-$intercepts of a function

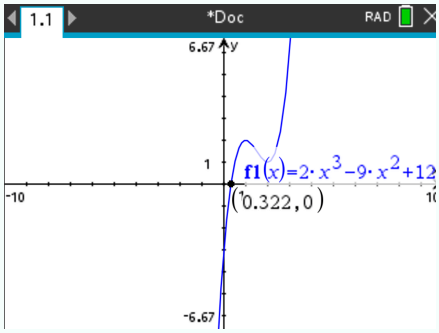
Suppose you want to know the $x-$intercept of the following function:

$$f(x) = 2x^3 - 9x^2 + 12x - 3$$

Recall that we want to compute the value of x when $f(x) = 0$.

① Enter the function, and display its graph.

② Press menu and select Analyze Graph > Zero. Select lower and upper bounds the widest possible. Press enter .



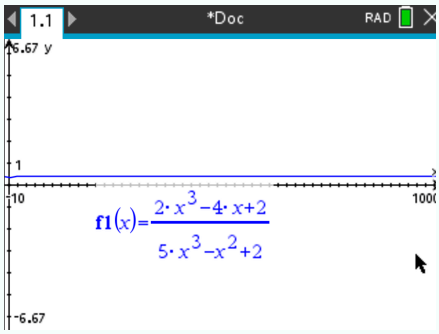
The result **X=0.322** should be displayed within the coordinate of the point.

2.4.4 Find a horizontal asymptote of a function

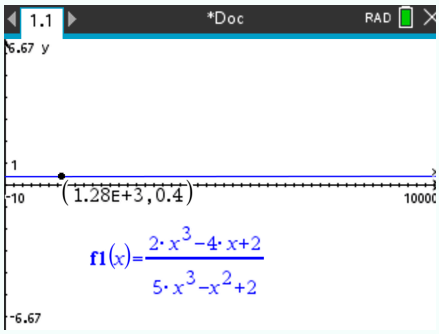
Suppose you want to see the horizontal asymptote of the following function when x goes to ∞ :

$$\frac{2x^3 - 4x + 2}{5x^3 - x^2 + 2}$$

① Enter the function and display huge values of x, like **Xmax=10000**:



② Press menu , select Analyze Graph > Maximum and take the y value of the maximum as the value of the horitzontal asymptote:



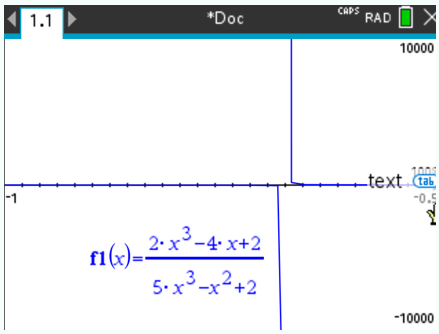
The result should be **Y=0.4** (rounded).

2.4.5 Find a vertical asymptote of a function

Suppose you want to see a vertical asymptote of the following function:

$$\frac{2x^3 - 4x + 2}{5x^3 - x^2 + 2}$$

① Enter the function and display $x-$values near the vertical asymptote and big $y-$values, here **Xmin=-1**, **Xmax=-0.5**, **Ymin=-10000** and **Ymax=10000**:



② Use the maximum and minimum to find the x-value of each vertical asymptote. The $x-$values should be **X=-0.678** at the left and **X=-0.674** at the right (rounded).

③ Take the average of the two values.

The result should be $x = 0.676$ (rounded).

> Depending on how precise you want the result to be, you will have to zoom in more, and move again the cursor

Here, by choosing **Xmin=-0.7** and **Xmax=-0.6** we get the result $x = 0.67613$ (rounded).

2.4.6 Find the intersection of two curves

Suppose you want to know one of the intersections of the graphs of the following functions:

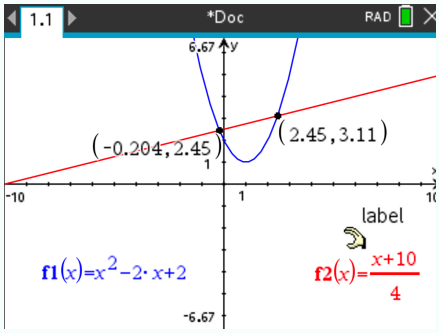
$$f(x) = x^2 - 2x + 2 \qquad g(x) = \frac{x + 10}{4}.$$

① Enter the two functions and graph them in order to see the intersection in question.

② Press menu and select Analyze Graph > Intersection.

③ Select right and left bound according to the intersection you want to know between the two intersections (right or left).

④ Press enter. The intersection point coordinates are displayed:



Press enter

The intersection points should be **(-0.204,2.45)** (left intersection point) and **(2.45,3.11)** (right intersection point) (rounded).